

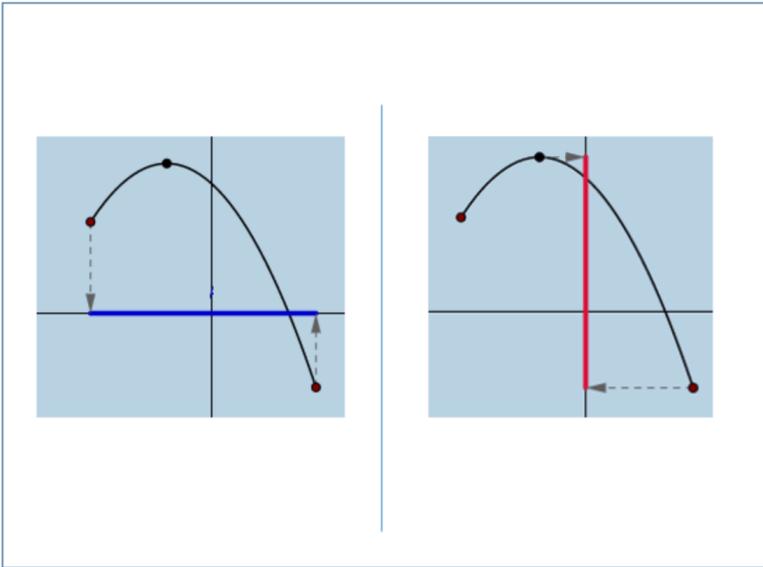
<https://sites.google.com/fcpsschools.net/justicehsibmaths/home>

To the new members of the IB Applications Family and returning members next year:

I can only imagine how much everyone is itching to get back a bit of normalcy with their scholarly endeavors. Come August, we will hit the ground running, straight into the glorious world that is IB Mathematics. To make sure that everyone in this family is equipped with the correct pair of running shoes, both HL and SL have the option of working through their respective packets which will represent the exact topics these classes will begin with on day one. These packets are for preemptive practice and will not be graded, but those that at least make an attempt will find themselves 100% familiar with what will be given in class.

Both the HL and SL classes will have access to our [IB Applications Google Site](#) and [YouTube Channel](#) for notes, videos and other materials which will be continuously updated throughout the summer. For any questions or assistance, I can be reached at htrussom@fcps.edu. For those returning for the year two class, I will make myself available over the summer for help with your Internal Assessments as promised—keep an eye out for announcements on Google Classroom.

Linear Modelling, Domain & Range



x	y
0	2
3	4
-3	-2
2	4

Find the Domain
and Range

{(,), (,), (,), (,)}

Domain: { , , , } Range: { , , , }

Equations of a Straight Line

Gradient-Intercept Form (useful when given a _____)

Point-Gradient Form (useful when given _____)

Standard Form (useful for _____)

Converting to Standard Form

Write the equation of the line $y = \frac{2}{3}x - 7$ in *standard form*.

Write the equation of the line $(y - 4) = \frac{3}{4}(x - 2)$ in *standard form*.

1a. [2 marks]

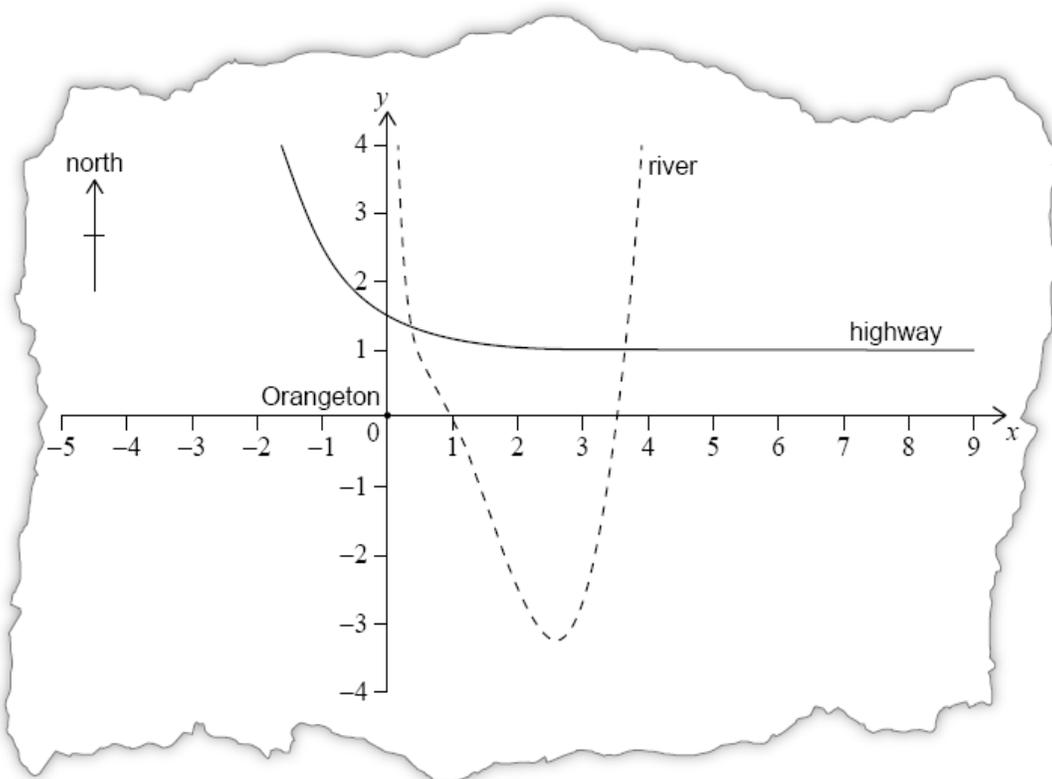
Consider the function $f(x) = x^3 - 5x^2 + 6x - 3 + \frac{1}{x}$, $x > 0$

Find the value of $f(x)$ when $x = \frac{1}{2}$.

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1b. [3 marks]

The function $f(x) = x^3 - 5x^2 + 6x - 3 + \frac{1}{x}$, $x > 0$, models the path of a river, as shown on the following map, where both axes represent distance and are measured in kilometres. On the same map, the location of a highway is defined by the function $g(x) = 0.5(3)^{-x} + 1$.



The origin, $O(0, 0)$, is the location of the centre of a town called Orangeton.

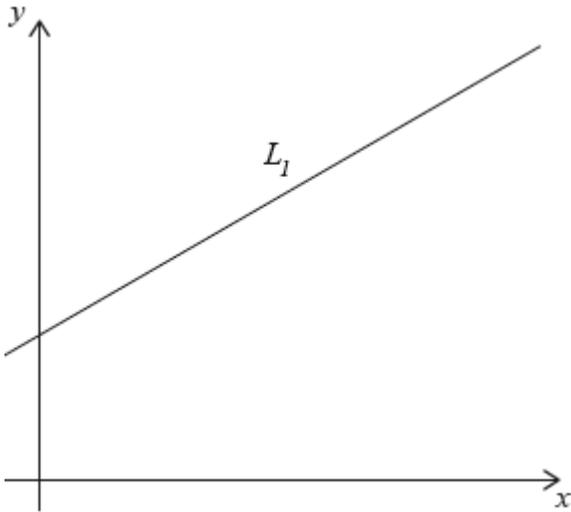
A straight footpath, P , is built to connect the centre of Orangeton to the river at the point where $x = \frac{1}{2}$.

Find the function, $P(x)$, that would define this footpath on the map.

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2a. [2 marks]

The line L_1 has equation $2y - x - 7 = 0$ and is shown on the diagram.



The point A has coordinates $(1, 4)$.

Show that A lies on L_1 .

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2b. [2 marks]

The point C has coordinates $(5, 12)$. M is the midpoint of AC.

Find the coordinates of M.

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2c. [2 marks]

Find the length of AC.

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3a. [3 marks]

Juan pays 8.75 euros (EUR) for a single movie ticket. The total amount Juan pays for movie tickets in a year can be modelled by

$$y = 8.75x$$

where x represents the number of tickets purchased per year and y represents the total amount, in EUR, paid per year.

Last year Juan spent less than 60 EUR.

Determine the maximum number of movie tickets Juan purchased last year.

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3b. [1 mark]

Maureen buys an annual movie ticket discount card for 50 EUR and then pays 2.50 EUR for each movie ticket.

Write down an equation in terms of x and y , using Maureen's information.

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3c. [2 marks]

During this year, Juan and Maureen will **each** buy the same number of tickets and will each pay the same total amount of money.

Find the number of tickets Juan will buy this year.

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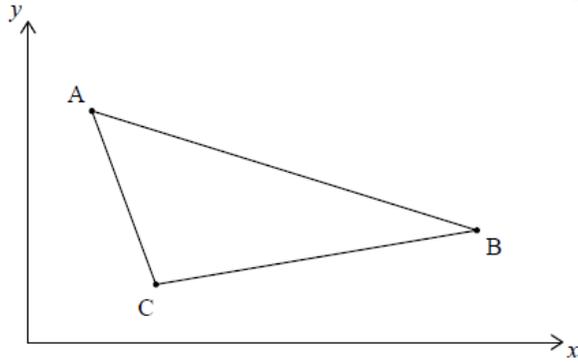
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4a. [2 marks]

The diagram shows a triangle defined by the points A(3 , 9), B(15 , 6) and C(5 , 3).

diagram not to scale



Calculate the gradient of the line AC.

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4b. [2 marks]

Determine, giving a reason, whether angle $\hat{A}CB$ is a right angle.

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4c. [2 marks]

The straight line, L , is parallel to BC and passes through A.

Find the equation of L .

Give your answer in the form $ax + by + d = 0$, where a , b and d are integers.

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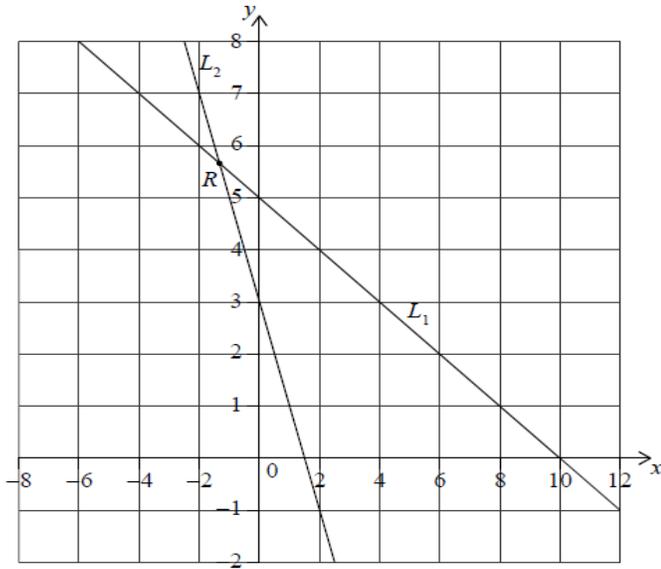
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5a. [2 marks]

Consider the straight lines L_1 and L_2 . R is the point of intersection of these lines.



The equation of line L_1 is $y = ax + 5$.

Find the value of a .

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5b. [2 marks]

The equation of line L_2 is $y = -2x + 3$.

Find the coordinates of R .

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5c. [2 marks]

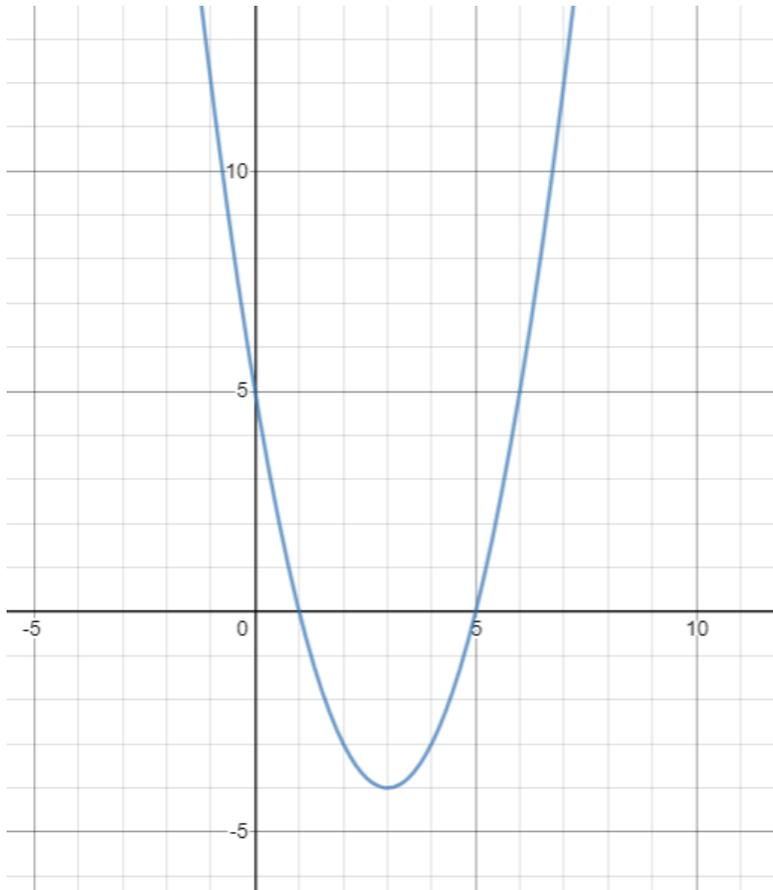
Line L_3 is parallel to line L_2 and passes through the point $(2, 3)$.

Find the equation of line L_3 . Give your answer in the form $y = mx + c$.

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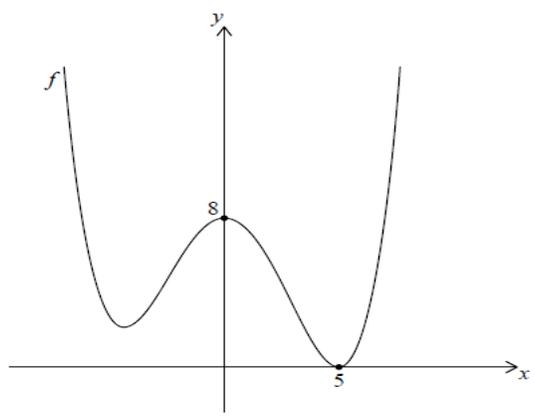
Transformations & Quadratic Functions

Rules for Transformation of Linear Functions		
Transformation	Function	Description
	$f(x + h)$	
	$f(x - h)$	
	$f(x) + k$	
	$f(x) - k$	
	$-f(x)$	
	$f(-x)$	
	$a f(x), a > 1$	
	$a f(x), 0 < a < 1$	
	$f(ax), a > 1$	
	$f(ax), 0 < a < 1$	



1a. [1 mark]

The following diagram shows part of the graph of f with x -intercept $(5, 0)$ and y -intercept $(0, 8)$.



Find the y -intercept of the graph of $f(x) + 3$.

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1b. [2 marks]

Find the y -intercept of the graph of $f(4x)$.

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1c. [2 marks]

Find the x -intercept of the graph of $f(2x)$.

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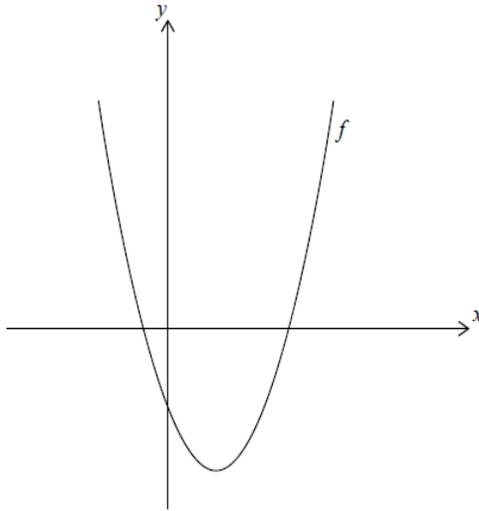
1d. [2 marks]

Describe the transformation $f(x + 1)$.

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2a. [5 marks]

Let $f(x) = x^2 - 4x - 5$. The following diagram shows part of the graph of f .



Find the x -intercepts of the graph of f .

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2b. [2 marks]

Find the equation of the axis of symmetry of the graph of f .

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2c. [1 mark]

The function can be written in the form $f(x) = (x - h)^2 + k$

Write down the value of h .

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2d. [3 marks]

Find the value of k .

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2e. [5 marks]

The graph of a second function, g , is obtained by a reflection of the graph of f in the y -axis, followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

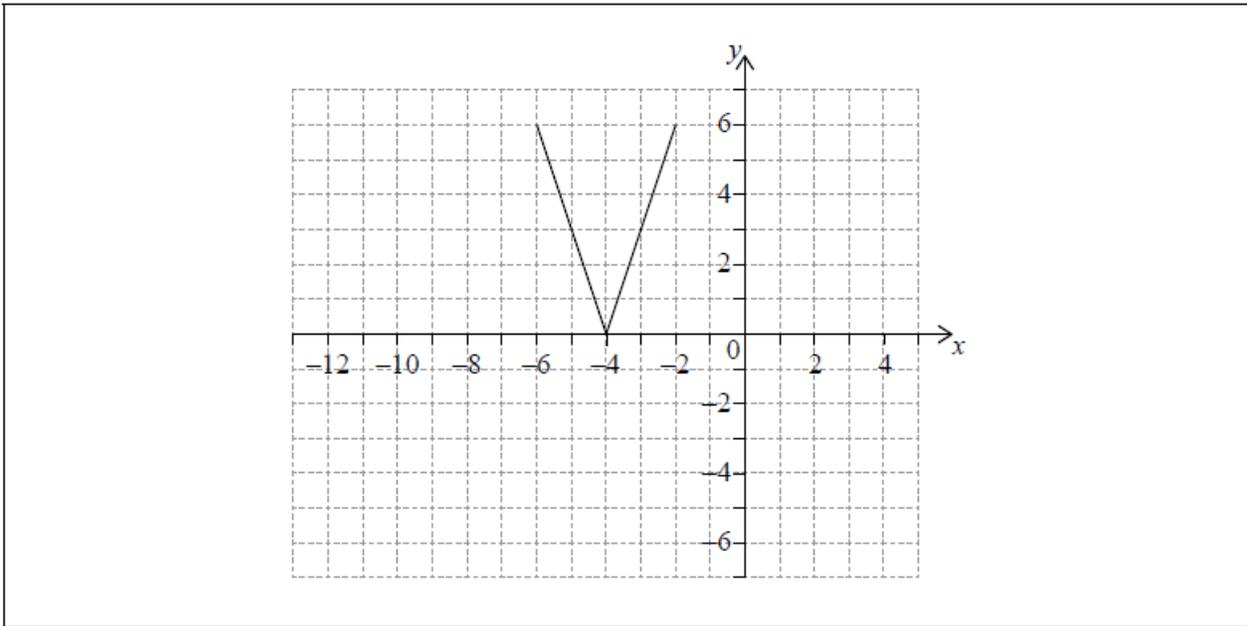
Find the coordinates of the vertex of the graph of g .

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3a. [2 marks]

The following diagram shows the graph of a function $y = f(x)$, for $-6 \leq x \leq -2$

The points $(-6, 6)$ and $(-2, 6)$ lie on the graph of f . There is a minimum point at $(-4, 0)$



Write down the range of f .

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3b. [2 marks]

Let $g(x) = f(x - 5)$.

On the grid above, sketch the graph of g .

3c. [2 marks]

Write down the domain of g .

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4a. [3 marks]

Let f and g be functions such that $g(x) = 2f(x + 1) + 5$.

The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k , followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k ;

(ii) p ;

(iii) q .

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4b. [3 marks]

Let $h(x) = -g(3x)$. The point A(6, 5) on the graph of g is mapped to the point A' on the graph of h . Find A' .

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5a. [2 marks]

Consider the graph of the function $f(x) = a(x + 10)^2 + 15$, $x \in \mathbb{R}$

Write down the coordinates of the vertex.

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5b. [2 marks]

The graph of f has a y -intercept at -20 . Find a .

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5c. [2 marks]

Point P(8, b) lies on the graph of f . Find b .

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6a. [2 marks]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L , intersects the graph of f at $x = -1$ and $x = 3$.

The equation of the axis of symmetry is $x = p$. Find p .

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6b. [2 marks]

Hence, show that $a = 2$.

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6c. [3 marks]

The equation of L is $y = 5$. Find the value of c .

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7a. [2 marks]

Let $f(x) = x^2 - 4x + 5$

Find the equation of the axis of symmetry of the graph of f .

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7b. [4 marks]

The function can also be expressed in the form $f(x) = (x - h)^2 + k$

- (i) Write down the value of h
- (ii) Find the value of k

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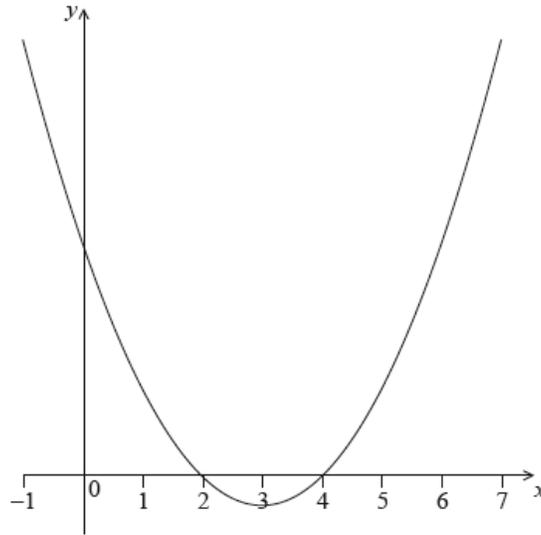
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8a. [2 marks]

The following diagram shows part of the graph of a quadratic function f .



The vertex is at $(3, -1)$ and the x -intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$

Write down the value of h and of k

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8b. [2 marks]

The function can also be written in the form $f(x) = (x - a)(x - b)$.

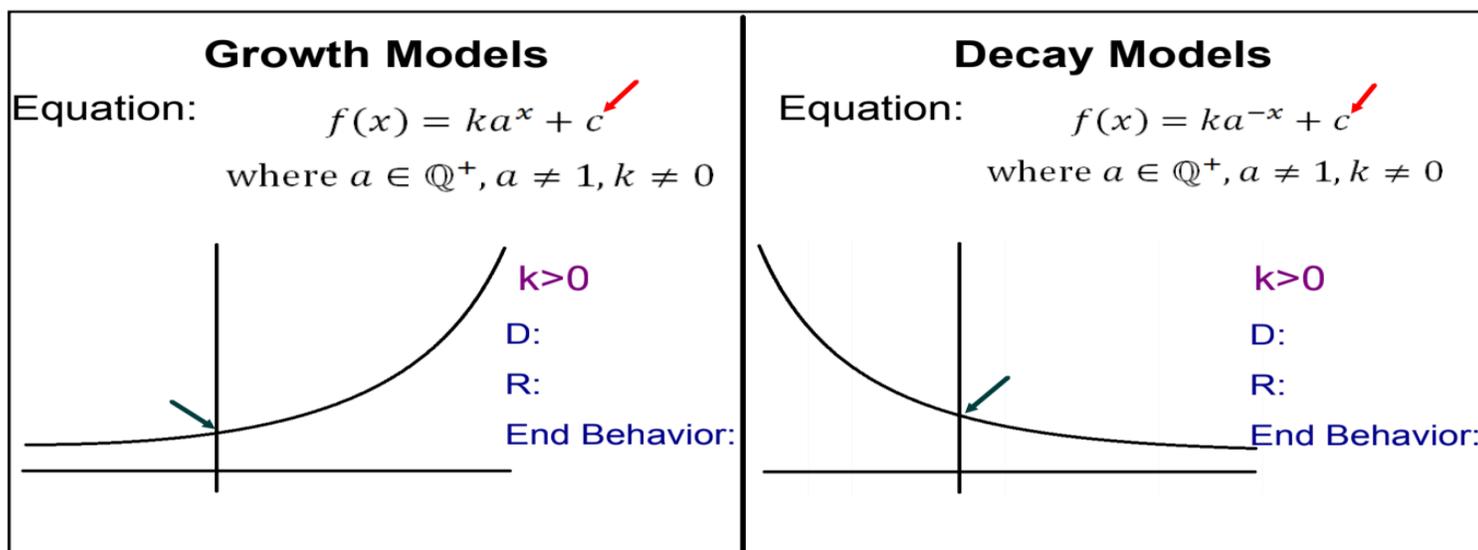
Write down the value of a and of b

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8c. [2 marks]

Find the y -intercept.

Exponential & Logarithmic Functions



Logarithmic Properties	
Product Rule	$\log_a(xy) =$
Quotient Rule	$\log_a\left(\frac{x}{y}\right) =$
Power Rule	$\log_a x^p =$
Change of Base Rule	$\log_a x =$
Equality Rule	If $\log_a x = \log_a y$ then

Solving exponential equations with logs:

$16^{3x} = 8$	$4^{x+2} - 1 = 14$	$4e^{2x} + 1 = 17$
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3a. [2 marks]

Little Green island originally had no turtles. After 55 turtles were introduced to the island, their population is modelled by

$$N(t) = a \times 2^{-t} + 10, \quad t \geq 0,$$

where a is a constant and t is the time in years since the turtles were introduced.

Find the value of a .

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3b. [2 marks]

Find the time, in years, for the population to decrease to 20 turtles.

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3c. [2 marks]

There is a number m beyond which the turtle population will not decrease.

Find the value of m . Justify your answer.

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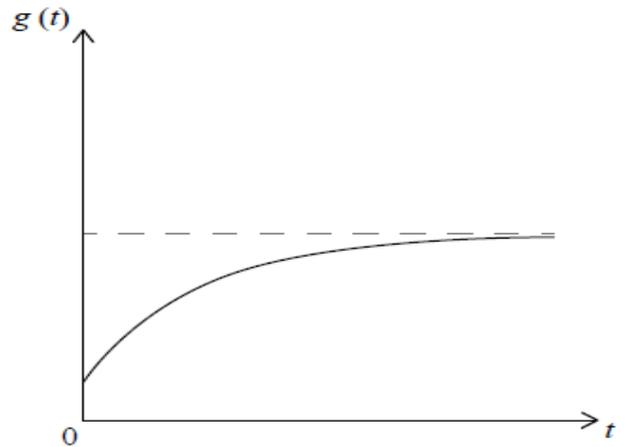
4a. [2 marks]

The amount of yeast, g grams, in a sugar solution can be modelled by the function,

$$g(t) = 10 - k(c)^t \text{ for } t \geq 0$$

where t is the time in minutes.

The graph of $g(t)$ is shown:



The initial amount of yeast in this solution is 2 grams.

Find the value of k .

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4b. [3 marks]

The amount of yeast in this solution after 3 minutes is 9 grams.

Find the value of c .

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4c. [1 mark]

Write down the maximum amount of yeast in this solution.

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5a. [2 marks]

The following function models the growth of a bacteria population in an experiment,

$$P(t) = A \times 2^t, t \geq 0$$

where A is a constant and t is the time, in hours, since the experiment began.

Four hours after the experiment began, the bacteria population is 6400.

Find the value of A .

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5b. [1 mark]

Interpret what A represents in this context.

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5c. [3 marks]

Find the time since the experiment began for the bacteria population to be equal to 40.

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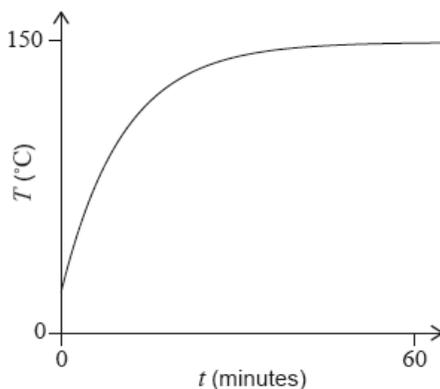
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6a. [1 mark]

Sejah placed a baking tin, that contained cake mix, in a preheated oven in order to bake a cake. The temperature in the centre of the cake mix, T , in degrees Celsius ($^{\circ}\text{C}$) is given by

$$T(t) = 150 - a \times (1.1)^{-t}$$

where t is the time, in minutes, since the baking tin was placed in the oven. The graph of T is shown in the following diagram.



Write down what the value of 150 represents in the context of the question.

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6b. [2 marks]

The temperature in the centre of the cake mix was 18°C when placed in the oven.

Find the value of a .

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6c. [3 marks]

The baking tin is removed from the oven 15 minutes after the temperature in the centre of the cake mix has reached 130°C .

Find the total time that the baking tin is in the oven.

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7a. [1 mark]

Jose takes medication. After t minutes, the concentration of medication left in his bloodstream is given by $A(t) = 10(0.5)^{0.014t}$, where A is in milligrams per litre.

Write down $A(0)$.

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7b. [2 marks]

Find the concentration of medication left in his bloodstream after 50 minutes.

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7c. [5 marks]

At 13:00, when there is no medication in Jose's bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again?

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